B.Sc. V SEMESTER PHYSICS

Constrained Motion:. **Constrained motion** is a term that describes a situation where a body is allowed to move in one direction while being restricted in all other directions. This means the object's motion is limited or constrained. A common example of this is a ceiling fan's circular motion.

Categories of Constrained Motion: Constrained motion can be classified into three main categories:

1. Fully constrained motion
2. Partially or successfully constrained motion
3. Incompletely constrained motion

1. Fully Constrained Motion:

- In a fully constrained motion, the pair's movement is restricted to a single direction, regardless of the direction of the applied force.
- One can visualize this with the example of a rectangular shaft moving in a single direction within a rectangular hole.
- The shaft cannot rotate or move in any other direction.

Example: Square bar in a square hole & Shaft with a collar at each end in the circular hole.

Partially or Successfully Constrained Motion:

- In a partially constrained motion, the motion can occur in more than one direction when no external force is applied.
- This type of motion is also referred to as successfully constrained motion.
- A footstep bearing that moves in a single direction when an external force is applied is an example of this type of motion.
- **Example:** The motion of the shaft in a footstep bearing becomes successfully constrained motion when a compressive load is applied.

Incompletely Constrained Motion:

- In an incompletely constrained motion, the pair's movement can take place in more than one direction.
- An example of this type of motion is a circular shaft moving within a circular hole.
- **Example:** Circular shaft in a circular hole, as it may either rotate or slide in a hole. Both motions have no relationship with other.

Lagrange formalism:

- In classical mechanics the particle equations of motion can be obtained from the Lagrange equation:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = 0$$

- The Lagrangian in classical mechanics is given by:

$$L = T - V = E_{kinetic} - E_{potential}$$

Newtonian mechanics:

- Newtonian mechanics**,** developed by Sir Isaac Newton, is based on his three laws of motion and the concept of forces.
- It describes the motion of objects in terms of their mass, acceleration, and the forces acting upon them.

Lagrangian mechanics:

- Lagrangian mechanics is based on the principle of least action.
- It describes the motion of objects in terms of kinetic and potential energies, and it uses generalized coordinates to describe the configuration of a system.
- The equations of motion in Lagrangian mechanics are derived from a single scalar function called the Lagrangian, which encapsulates the dynamics of the system.

One key difference between the two frameworks is that Lagrangian mechanics provides a more elegant and powerful formalism for dealing with complex systems and constraints. It also has a strong connection to the principle of least action, which is a fundamental concept in physics.

In summary, while both Newtonian and Lagrangian mechanics are used to describe the motion of objects, Lagrangian mechanics offers a more generalized and mathematically elegant approach to understanding the dynamics of physical systems

Hamiltonian:

- In quantum mechanics, the **Hamiltonian** of a system is an operator corresponding to the total energy of that system, including both kinetic energy and potential energy.
- The Hamiltonian is named after William Rowan Hamilton who developed a revolutionary reformulation of Newtonian Mechanics, known as Hamiltonian mechanics, which was historically important to the development of quantum physics.

Hamiltonian formulation

By the Legendre transform, i.e.

$$\mathcal{H}(\rho, S) = \sup_{\dot{\rho}} \dot{\rho}^T S - \mathcal{L}(\rho, \dot{\rho}),$$

then the geodesics satisfies the Hamiltonian system

$$\frac{d}{dt}\begin{pmatrix} \rho \\ S \end{pmatrix} = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix} \begin{pmatrix} \frac{\partial}{\partial \rho}\mathcal{H} \\ \frac{\partial}{\partial S}\mathcal{H} \end{pmatrix},$$

where

$$\mathcal{H}(\rho, S) = \frac{1}{2} S^T L(\rho) S.$$

Comparison of Hamiltonian & Lagrangian formulation:

Formulation	Hamiltonian	Lagrangian
Approach	Schrödinger Eq. $H\|E_n\rangle = E_n\|E_n\rangle$	Path Integral $\langle O \rangle = \frac{\int [d\varphi] O[\varphi] \exp(-S[\varphi]/\hbar)}{\int [d\varphi] \exp(-S[\varphi]/\hbar)}$
Algorithm	Series expansion, variational, Runge-Kutta ...	MC simulation
Advantage	Both the ground state, and the excited states can be computed.	It generates the most important configs. for the measurements.
Disadvantage	Analytical methods are too tedious for many body systems; Runge-Kutta works only in 1-D.	It is difficult to study the excited states, and finite density QCD.

Central Force: The central force in classical mechanics is defined as the force that is acting on an object which is directed along the line joining the object and the origin. The magnitude of the central force depends only on the distance between the object and the centre. Examples of central forces are gravitational force, electrostatic forces, and spring force.

The study of the central forces and their characteristics plays a very crucial role in understanding the following motions in the universe.

- The motion of planets around the sun
- The motion of satellite around the earth
- The motion between the charge particle

Following are the theorems that relate central force with angular momentum:

Theorem 1: *For an object to have its angular momentum conserved, the object should be subjected only to the central force.*

Theorem 2: *For an object to have its motion on a plane, the object should be subjected only to the central force.*

Characteristics of Central Force

- Central forces are long-range forces, depending on the distance of separation, its magnitude decreases as the separation increases.
- It acts along a line joining the centre of two bodies, that's why these are called central forces.

PART –B: INTRODUCTION TO STATISTIAL MECHANICS

Microstate: In physics, a microstate is defined as the arrangement of each molecule in the system at a single instant.

Macrostate: A macrostate is defined by the macroscopic properties of the system, such as temperature, pressure, volume, etc. For each macrostate, there are many microstates which result in the same macrostate

Statistical Mechanics: Statistical mechanics is the branch of physics in which we study the total behaviour of particles in the system.

Statistical Thermodynamics: Statistical Thermodynamics is a theory that provides a quantitative link between the properties of microscopic particles and of the behaviour of the macroscopic system.

- Statistical thermodynamics gives a more profound comprehension to in any case to some degree hazy ideas such as

- Thermodynamics equilibrium
- Concept of Free energy
- Entropy
- Laws of thermodynamics

Some of the applications of statistical mechanics can be as follows:

- Gibb presented his experimental evidence in thermodynamics by applying the concept of statistical mechanics.

- The principle application of statistical mechanics can be found while studying the Maxwell distribution of velocity law

Ensemble in Statistical Mechanics: In thermodynamics, the world is constantly separated into a system and its environmental elements.

Classification of Ensembles

- The systems in an ensemble are regularly not all in the equivalent microstate or macrostate.
- However, every one of them is associated similarly with their environmental elements.
- Subsequently, ensembles can be characterised or classified by the way their systems associate with their environmental elements.

✓ Microcanonical ensemble
✓ Canonical ensemble
✓ Isothermal – isobaric ensemble
✓ Grand canonical ensemble

Statistical Distribution Law:

1. Maxwell –Boltzmann Statistics
   Distribution of identical particles which are widely separated. Example: gas molecules

$$n(E) = A\, g(E) e^{-E/k_B T}$$

2. Bose Einstein Distribution
   For identical particles with spin 0 or integer/boson. Example: photon, phonon.

$$n(E) = A \frac{g(E)}{e^{\alpha} e^{-E/k_B T} - 1}$$

3. Fermi Diract Distribution
   For particles with odd half-integer spin/fermion (1/2, 3/2, ...). Example: electron

$$n(E) = A \frac{g(E)}{e^{\alpha} e^{-E/k_B T} + 1}$$

Boltzmann's canonical distribution law: The Boltzmann distribution law states that the probability of finding the molecule in a particular energy state varies exponentially as the energy.

The Energy Distribution Function

(Number Density) $n_V(E)$:

- This is defined so that $n_V(E)\,dE \equiv$ the number of molecules per unit volume with energy between E and E + dE

$$n_V(\mathcal{E}) = n_0 e^{-\mathcal{E}/kT}$$

The Canonical Probability Function P(E):

- This is defined so that $P(E)\,dE \equiv$ the probability to find a particular molecule between E and E + dE

$$P(E) = \frac{e^{-E/kT}}{Z}$$