

B.Sc. I SEMESTER PHYSICS

PART-A BASIC MATHEMATICAL PHYSICS

Physical quantity:

There are two types of physical quantities in vector algebra namely scalar and vector.

- **Scalar quantity:** Any physical quantity which has only magnitude but no direction is declared to be a scalar quantity.
- **Vector quantity:** Any physical quantity which has both magnitude and direction is stated to be a vector quantity.

Vector Algebra:

- Vector algebra is one of the essential topics of algebra. It studies the algebra of vector quantities. It is denoted as \vec{v} .

Types of Vectors in Vector Algebra

- **Zero or Null Vectors:** A vector, whose initial and terminal locations coincide is declared a zero or null vector. This means a zero vector cannot be assigned a definite direction as it possesses zero magnitudes. Thus, the modulus of the null vector is zero and it is denoted by 0 or $\vec{0}$.
- **Unit Vector:** A vector whose magnitude is of unit length is designated as a unit vector. If \vec{x} is a vector whose magnitude is x, then unit vector \hat{x} in the direction of x is denoted by \hat{x} .
- **Like Vectors:** Vectors are assumed to be like when they have an identical direction but their magnitude is different.
- **Unlike Vectors:** when Vectors possess opposite directions and different magnitudes then they are stated to be unlike vectors.
- **Equality of Vectors:** Two vectors \vec{x} and \vec{y} are assumed to be equal if they have the same length and same direction.
- **Coinitial Vectors:** Vectors possessing the same original point are called coinital vectors.
- **Collinear Vectors:** The vectors which are parallel to the same straight line are supposed to be collinear vectors.
- **Coplanar vectors:** Three or more vectors are declared to be coplanar when they are parallel to the same plane otherwise, they are declared to be non-coplanar vectors.
- **Position Vector:** If a point O is fixed as the origin in space and P is any point, then **OP** is called the position vector of P concerning O.

Vector Algebra Operations: Vector algebra constitutes various vector rules and operations including vector addition, multiplication of vectors, dot product, cross products and more. Let us look at each of them in detail with the formulas, rules, properties and more.

Addition of Vectors:

The addition of two vectors \vec{x} and \vec{y} is denoted by $\vec{x} + \vec{y}$ and it is known as the resultant of \vec{x} and \vec{y} . There are various methods for the addition of vectors.

Triangle Law: If \vec{x} and \vec{y} lie along two consecutive sides of a triangle, then the third side represents the sum of the vectors \vec{x} and \vec{y} i.e. $\vec{x} + \vec{y}$.

Parallelogram Law: If \vec{x} and \vec{y} are the two adjacent sides of a parallelogram then their sum is represented by the diagonal of the parallelogram i.e. diagonal of the parallelogram represents the vector $\vec{x} + \vec{y}$.

The Gradient, Divergence, and Curl:

- The gradient of a scalar function $f: \mathbb{R}^n \rightarrow \mathbb{R}$ is a vector field of partial derivatives. In \mathbb{R}^2 , we have: $\nabla f = \langle \partial x / \partial f, \partial y / \partial f \rangle$.

The divergence:

Let $F: \mathbb{R}^3 \rightarrow \mathbb{R}^3 = \langle F_x, F_y, F_z \rangle$ be a vector field. Consider now a small box-like region, R , with surface, S , on the cartesian grid, with sides of length Δx , Δy , and Δz with (x, y, z) being one corner. The outward pointing unit normal are $\pm i^\wedge$, $\pm j^\wedge$, and $\pm k^\wedge$.

The curl

Before considering the curl for $n=3$, we derive a related quantity in $n=2$. The "curl" will be a measure of the microscopic circulation of a vector field.

Coordinate System

A coordinate system in geometry is a system that uses one or more numbers, or coordinates, to define the position of points uniquely

Types of Coordinate System

The important types of coordinate systems are as follows:

- Cartesian coordinate system
- Spherical coordinate system
- Cylindrical coordinate system
- Rectangular coordinate system
- 3D coordinate system
- Polar coordinate system

Cartesian Coordinate System: A Cartesian Coordinate System in a plane is a coordinate system in which a pair of two numerical coordinates uniquely specify each point on the plane.

Spherical Coordinate System: A spherical coordinate system is a three-dimensional coordinate system in which the position of a point is represented by three numbers: radial distance, polar angles, and azimuthal angle.

Cylindrical Coordinate System: The location of a third-dimensional point is described in a cylindrical coordinates system. The coordinates describe two distances and one angle. The normal polar coordinate is denoted by r .

Rectangular Coordinate System: Two real number lines connect at a right angle to form the rectangular coordinate system. The horizontal number line is called the x -axis, while the vertical number line is called the y -axis.

3D Coordinate System: A three-dimensional coordinate system of three coordinates determines a point's position in space.

Tensor algebra

- The tensor algebra is the algebra of tensor on V with multiplication being the tensor product.

- It is the free algebra on V , in the sense of being left joint to the forgetful functor from algebras to vector spaces:

Tensor Analysis

The mathematical branch which generally comprises all the relationships and laws which remain constant and valid irrespective of the coordinate system using which the quantities have been specified is called tensor analysis

Application of Tensors

- In the introduction to tensor analysis that Einstein has used it to derive the theory of relativity.
- Tensors have a vast application in physics and mathematical geometry.

PART B: NEWTONIAN MECHANICS & WAVE MOTION

Elastic collision:

A collision in which there is absolutely no loss of kinetic energy is called elastic collision.

Examples

- When a ball at a billiard table hits another ball, it is an example of elastic collision.
- When you throw a ball on the ground and it bounces back to your hand, there is no net change in the kinetic energy, and hence, it is an elastic collision.

Characteristics:

1. The linear momentum is conserved.
2. Total energy of the system is conserved.
3. Kinetic energy is conserved.

Elastic Collision Formula

The Elastic Collision formula of momentum is given by: $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

The Elastic Collision formula of kinetic energy is given by:

$$(1/2) m_1u_1^2 + (1/2) m_2u_2^2 = (1/2) m_1v_1^2 + (1/2) m_2v_2^2$$

Inelastic collision:

A collision in which there is some loss of kinetic energy is called inelastic collision.

Examples

- The accident between two cars or any other vehicles.
- When a soft mud ball is thrown against the wall, it will stick to the wall
- A ball falling from a certain altitude and unable to return to its original bounce.

Characteristics:

1. Linear momentum is conserved.
2. Total energy is conserved.
3. K E is not conserved.

Inelastic Collision Formula

When two objects collide with each other under inelastic conditions, the final velocity of the object can be obtained as;

$$V = (M_1 V_1 + M_2 V_2) / (M_1 + M_2)$$

Moment of Inertia:

The moment of inertia is defined as, the quantity expressed by the body resisting angular acceleration which is the sum of the product of the mass of every particle with its square of the distance from the axis of rotation.

The moment of inertia depends on the following factors:

- The density of the material
- Shape and size of the body
- Axis of rotation (distribution of mass relative to the axis)

Motion of Satellites:

- Satellites follow a circular route as they orbit the planet which is known as Motion of Satellites.
- The satellite's velocity is shown by the tangent to this circular route, whereas acceleration is in the direction of the circle's centre. Between the satellite and the Earth, a number of factors interact to cause them to orbit in a circular pattern.

Critical Velocity

- Critical velocity is defined as the minimum velocity required to put the motion of satellites in a stable circular orbit around any celestial object.
- It is also known as orbital velocity and can be denoted as v_c or v_o

i.e., $F_{CP} = F_g \Rightarrow mv_c^2 / r = GMm / r^2$

\therefore critical velocity, $v_c = \sqrt{GM / r}$

Escape Velocity

- Escape velocity is defined as the minimum velocity required by the satellite to project it vertically outward in order to escape Earth's Gravitational field of influence. It is given by symbol v_e which can be given as

Escape Velocity, $v_e = \sqrt{2GM / r} = \sqrt{2} v_c$

Now, the relation between escape velocity and critical velocity can be given as $v_e = 2\sqrt{v_c}$

The minimum velocity needed by a satellite to escape the influence of the earth's gravitational field is around 11.6 km/s.

Kepler's law of planetary motion: There are three laws are as follows:

- (1) planets move in elliptical orbits with the Sun as a focus.
- (2) a planet covers the same area of space in the same amount of time no matter where it is in its orbit.
- (3) a planet's orbital period is proportional to the size of its orbit (its semi-major axis).

Wave Motion

Wave motion is the transfer of energy and momentum from one point of the medium to another point of the medium without the actual transport of matter between two points. Wave motion is classified in three different ways, and they are as follows:

- The medium of propagation
- The dimensions in which a wave propagates energy
- The energy transfers.

Characteristics of Wave Motion

- Energy and momentum are transferred from one point to another without any actual transfer of the particles of the medium.
- There is a regular phase difference between the particles of the medium because each particle receives disturbance a little later than its preceding particle.

Terminologies in Progressive Wave Motion

Amplitude (A): The amplitude of a wave is the maximum displacement of any particle of the medium from its equilibrium position.

Period (T): The period of a wave is the time taken by any particle of the medium to complete one vibration during a period (T).

Wavelength (λ): Wavelength is equal to the distance between two consecutive particles of the medium, which are in the same state of vibration. It is equal to the distance travelled by the wave by its time period (T).

Frequency (f): It is the number of vibrations made per second by any particles of the medium ($f = 1/T$).

Wave Velocity (v): It is the distance travelled by the wave in one second ($v = \lambda/T$).

- It is determined by the mechanical properties of the medium through which the wave propagates.